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Near-Optimal Three-Dimensional Air-to-Air Missile Guidance Against Maneuvering Target

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Closed-loop guidance of a medium-range air-to-air missile (AAM) against a maneuvering target is synthesized using a three-phase, near-optimal guidance scheme. A large period of the closed-loop guidance is performed using neighboring optimal control techniques about open-loop optimal solutions obtained by solving the associated minimum time to intercept trajectories. The first phase is the boost phase guidance where the normal acceleration limit may be active due to high lofting of the boost–sustain–coast AAM. The guidance in this phase accommodates only errors in the missile's state variables, the target maneuvering being neglected. The boost phase guidance involves guidance in the presence of active control constraints. The second phase is the midcourse guidance where both state perturbations and target maneuvers are considered. Comparisons are made between guidance with gain indexing performed with clock time and with performance index to go. Models of aggressive target and run-away targets were used and the guidance scheme performance is excellent. Three methods of optimal gain evaluation are also discussed. Performance augmentation is obtained by using a center of attainability as a pseudotarget that fairs into the actual target as time to go becomes zero. The final phase is the terminal guidance, which employs proportional navigation and its variants.

Introduction

THERE are three phases of missile flight and their characteristics are sufficiently different so as to require the use of more than one guidance scheme.[1] A typical flight would include boost, midcourse, and terminal guidance phases. The boost phase is from initial firing to the time when the boost motor shuts down (and possibly initiates staging). Then on, the missile continues in flight either under sustain power or coasting. The character of range/time optimal flight during this early boost phase is in general radically different from the midcourse phase. During terminal guidance the shortened time line generally requires autonomous guidance of the missile.

Although efficient intercept has been of interest at least since the time of Sir Francis Drake and his "manouevre board," it was the requirements of World War II that prompted serious study of the problem of guidance against a maneuvering target. Pure pursuit, command to line of sight (LOS), collision course and proportional navigation (PN) were some developments from this interest.[1] PN guidance was developed in the United States by Yuan,[2] Newell,[3] and Spitz.[4] Murtaugh and Criel[5] in their study closed the classic approach era to PN. Later studies demonstrated that PN guidance produced a control that was optimal, in a specified sense, for certain linear models.[6–9] By incorporating additional features in the system model, it was possible to extend classical PN and improve the end-game performance.[10] Even today, the guidance for most short-range air-to-air missiles (AAMs) is based on PN or one of its variants.

Much of the research in the area of midcourse guidance has employed reduced-order modeling to obtain near-optimal feedback laws.[11,12] Such models are based on the mathematical theory of singular perturbation; applications in the broad area of flight control and guidance were pioneered by Kelley,[13] Ardema,[14] and Calise.[15] Recently, it has been shown[16] that for vertical plane motions, the control histories produced by various reduced-order models do not provide very accurate approximations of the corresponding control for a point-mass model. It is expected that the three-dimensional flight would also exhibit these differences.

For this reason, the major purpose of this study has been to obtain a range/energy/time efficient guidance law for an interceptor missile modeled as a point mass in three-dimensional atmospheric flight[17,18] using the principles of optimal control theory. Extremal fields and neighboring extremal ideas[19] are used hand in hand with the nominal solutions to develop a closed-loop guidance algorithm.

The nominal optimal control problem solved[17] is one of maximizing range for a fixed time of flight. However, once the missile is launched, the real motive is no longer to maximize range at the given time. This can be seen clearly in the case of a target turn-away. Since the nominal is based on a head-on engagement, if the target turns away, the predicted intercept time and range will increase markedly. The nominal optimal control solutions used for reference is a reciprocal problem (via Mayer reciprocity), namely, the minimum-time-to-intercept problem.

This paper discusses a three-phase near-optimal guidance scheme of an AAM against a target capable of maneuvers in the horizontal plane. The idea can be extended to intercepting a target maneuvering in the vertical plane as well. The major contributions of this paper include 1) some simple ideas of optimal gain evaluation, 2) optimal closed-loop missile guidance in the presence of control constraints, 3) performance augmentation using a center of attainability as the pseudotarget, and 4) that neighboring optimal control can be successfully used for AAM guidance.

Open-Loop Solutions

The solutions to the nominal optimal control problem[17] resulted in defining the boundary of attainability of the missile in the horizontal projection (x–y) for fixed final time with the other states fixed or unprescribed. The following assumptions are made: 1) point-mass model; 2) flat Earth; 3) three-dimensional missile trajectory; 4) air density that varies with altitude; 5) drag that is a function of altitude, Mach number, and control effort; and 6) thrust and weight histories that are predetermined functions of time.[17]

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The equations of motion for the three-dimensional point-mass model[17] of the missile used for generating the open-loop optimal trajectories are as follows:

$$\begin{aligned}\dot{x} &= V\cos\gamma\cos\chi\\ \dot{y} &= V\cos\gamma\sin\chi\\ \dot{h} &= V\sin\gamma\\ \dot{E} &= [T - D(h, M, n)](V/W)\\ \dot{\gamma} &= (n_v - \cos\gamma)\frac{g}{V}\\ \dot{\chi} &= \frac{n_h}{\cos\gamma}\,\frac{g}{V}\end{aligned} \tag{1}$$

where x is the down range of the missile, y the cross range, h the altitude, and E the specific energy. The velocity is obtained from the algebraic equation $V = \sqrt{2g(E-h)}$. The flight-path angle is γ, and χ denotes the heading angle; T is the thrust magnitude, D the aerodynamic drag, W the weight of the missile, and M the Mach number. The two controls are n_v, the vertical load factor, and n_h, the horizontal load factor. The resultant load factor is obtained from the algebraic expression $n = \sqrt{(n_v^2 + n_h^2)}$. Drag is a function[17] of altitude, Mach number, and load factor. The control variables are limited by two constraints:

Structural limit

$$n(t) \le n_{\max}$$

Aerodynamic limit

$$n(t) \le n_L \doteq C_{L,\max}(M)(qS/W)$$

where $n_{\max}$ is a constant, $C_{L,\max}(M)$ is the Mach-dependent maximum lift coefficient,[18] q is the dynamic pressure, and S is the characteristic area of the missile.

The nominal optimal control problem solved[17] is used to generate the x–y range charts, as shown in Fig. 1, indicating maximum range for various prescribed final time t_f. This was performed with boundary conditions as follows:

Initial conditions

$$x(t_0) = 0, \qquad y(t_0) = 0, \qquad h(t_0) = h_0$$

$$E(t_0) = E_0, \qquad \gamma(t_0) = 0, \qquad \chi(t_0) = 0$$

Final conditions

$$\boxed{\begin{aligned}x(t_f) &= x_{\max/\min}\\ y(t_f) &= y_{\text{fixed}}\end{aligned}} \quad \text{or} \quad \boxed{\begin{aligned}x(t_f) &= x_{\text{fixed}}\\ y(t_f) &= y_{\max/\min}\end{aligned}}$$

$$h(t_f) = h_f, \qquad E(t_f) \ge E_f, \qquad \gamma(t_f) = \gamma_{\text{free}}$$

$$\chi(t_f) = \chi_{\text{free}}$$

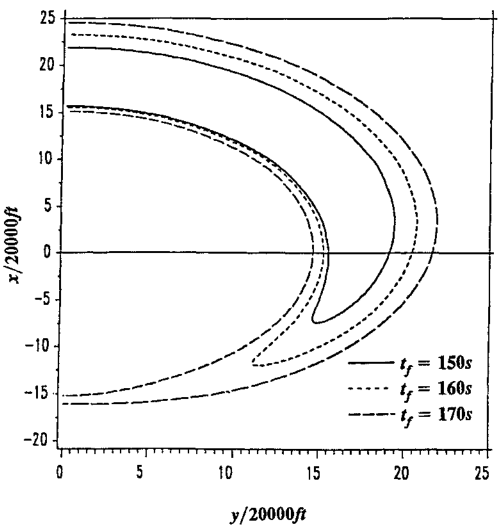

Fig. 1 Typical range chart (t_f = 150, 160, 170 s).

It is clear that if no prelaunch maneuvers are performed, then a four-parameter family (h_0, h_f, E_0, E_f) of such range charts are required to completely flood the state space with extremals. In the guidance schemes proposed in the following sections, it is assumed that the nominal maximum range solutions corresponding to a coarse (5000-ft) discretization of the four-parameter space are stored on the flight computer of the launching aircraft. The reference solutions stored are the minimum-time-to-intercept solutions, which is equivalent, by Mayer reciprocity, to the maximum-range solutions.

Once the target is viewed, the nominal intercept (x, y) coordinates can be obtained as follows. Discretized (x, y) coordinates obtained from the maximum-range boundary of the attainability sets and the corresponding final time t_f to reach these are stored in tabular form (x, y; t_f) in the flight computer. Given the current target position and an estimated intercept time t_f, the target position can be extrapolated linearly in the velocity direction. This gives an ordered pair (x_T, y_T). The value of t_f is iteratively adjusted so that (x_T, y_T) is within some prescribed tolerance of the point (x, y) interpolated from the tabular data. In short, the missile coordinates at final time must be equal to the extrapolated target position at final time. Once the nominal intercept coordinate is located, the closest nominal solution (i.e., minimizing 2-norm distance), defined as the reference trajectory, is used for closed-loop guidance.

Boost Phase Guidance

The initial maneuver performed by the AAM is a combination of lofting to reach high altitudes during the thrust phase and simultaneous horizontal plane maneuvers to align the missile with the predicted intercept point (PIP). The horizontal and vertical load factors have high magnitudes during this phase. The structural and aerodynamic limits of the missile constrain the control efforts. In this segment of the guidance scheme, the problem of near-optimal guidance with an active control constraint is considered. Neighboring optimal control in the presence of control and state constraints is typically a nontrivial problem.[20,21]

The possibility of an active normal acceleration limit during this phase was simulated via a nominal problem with an active structural limit. The resultant load factor for the nominal intercept problem, solved by using the software BOUNDSCO,[22] hits the constraint and remains on it for a finite period of time before leaving it. The nominal optimal control problem[17] uses n_v and n_h as the two control variables. The same problem and solution can be easily transformed with resultant load factor n and bank angle μ_b as the two new control variables. This eases the development of the neighboring optimal guidance scheme. The two new controls are related to the old ones by the formulas $n_v = n\cos\mu_b$ and $n_h = n\sin\mu_b$.

Figure 2 shows the nominal time history of the resultant load factor. Let t_1 be the time of entry of load factor n onto the control

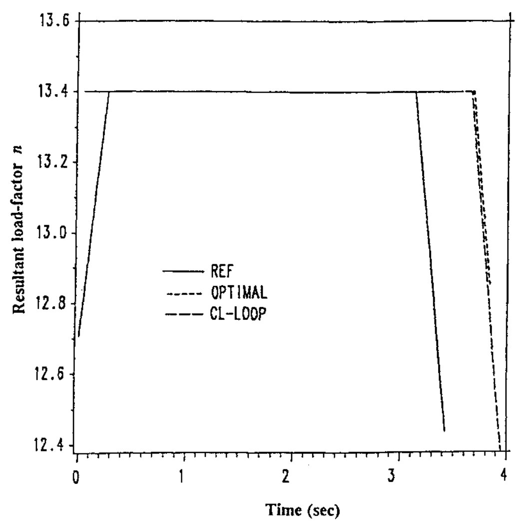

Fig. 2 Load factor history: boost phase.

constraint and let t_2 be the time of exit of the load factor from the control constraint. While developing gains for the closed-loop guidance, we assume the following:

1) Neighboring extremals have the same switching structure as regards to the control constraint and thrust switching.

2) The perturbations in state variables are "small."

The optimal gains required for the closed-loop guidance are obtained as follows: At some initial time t_0 a small positive perturbation is made in one of the states and the new optimal control intercept problem is solved via the two-point boundary value problem (TPBVP). The new state and control at t_0 is noted. This is repeated for a small negative perturbation. The gain at t_0 for the required control–state pair is evaluated using central difference as $\Delta control(t_0)/\Delta state(t_0)$. This is then performed for all other individual state perturbations. The procedure is repeated at different time nodes[23] to obtain time-varying gains.

A more efficient method to evaluate the optimal gains are as follows. At initial time t_0 a small perturbation is made in one of the states as before and the new intercept problem re-solved. The state and control values from the new TPBVP are stored at a number of sample points. The changes in controls and states from the nominal solution are recorded as a function of time. Next, a perturbation in the second state is made and the change in controls and states from nominal is recorded. This is continued for all six states and yields a set of time-varying linear equations that when solved gives the required gains.

The set of equations for the load factor control n are

$$\begin{bmatrix} \delta n_1(t) \\ \delta n_2(t) \\ \delta n_3(t) \\ \delta n_4(t) \\ \delta n_5(t) \\ \delta n_6(t) \end{bmatrix} = [A(t)]G_n^T(t) \tag{2}$$

where

$$[A(t)] = \begin{bmatrix} \delta x_1 & \delta y_1 & \delta h_1 & \delta E_1 & \delta \gamma_1 & \delta \chi_1 \\ \delta x_2 & \delta y_2 & \delta h_2 & \delta E_2 & \delta \gamma_2 & \delta \chi_2 \\ \delta x_3 & \delta y_3 & \delta h_3 & \delta E_3 & \delta \gamma_3 & \delta \chi_3 \\ \delta x_4 & \delta y_4 & \delta h_4 & \delta E_4 & \delta \gamma_4 & \delta \chi_4 \\ \delta x_5 & \delta y_5 & \delta h_5 & \delta E_5 & \delta \gamma_5 & \delta \chi_5 \\ \delta x_6 & \delta y_6 & \delta h_6 & \delta E_6 & \delta \gamma_6 & \delta \chi_6 \end{bmatrix}$$

$$G_n^T(t) = \begin{bmatrix} n_x(t) \\ n_y(t) \\ n_h(t) \\ n_E(t) \\ n_\gamma(t) \\ n_\chi(t) \end{bmatrix}$$

where the unknowns are $n_x(t) = \partial n/\partial x$, etc., corresponding to the elements of the row vector $G_n(t)$. Here, $\delta n_i(t)$ is the difference in load factor between the nominal and perturbed solutions with perturbation at initial time done to the ith state ($i = 1$ corresponds to down range, x, etc.) Another such set of six equations exists for the second control μ_b, which when solved gives the gain vector $G_\mu(t)$. The rows of the composite gain matrix $G(t)$ are given by $G_n(t)$ and $G_\mu(t)$.

The advantage of the second method of gain evaluation is that new optimal control problems need not be solved at different times. Inaccuracies arising from the solution of an ill-conditioned set of linear equations is a possible disadvantage. The fundamental matrix obtained by linearizing the original system about the reference trajectory and expressing in a closed-loop form is a first-order approximation of the transpose of the time-varying matrix A in Eq. (2). Theoretically, the fundamental matrix cannot be singular. Neglecting second-order errors, one can presume the nonsingularity of the above matrix if six independent perturbation vectors are used at initial time. Thus, one expects that, for sufficiently small initial perturbations, the matrix will not be singular. However, the matrix may be badly conditioned for inversion. A third approach, namely the conventional Riccati approach[24] to obtain feedback gains, was not implemented for this phase of the guidance because the constrained arc complicates the analysis.

The gains obtained by the second method are meaningful only from time t_0 to t_1^-, t_1^+ to t_2^-, and t_2^+ to t_f, where t_1^- is the earliest time at which the load factor control gets saturated due to state perturbations at initial time for gain evaluations, t_1^+ is the latest time at which the load factor control gets saturated due to state perturbations at initial time for gain evaluation, and t_2^- and t_2^+ correspond to the earliest and latest times for exit from the control constraint, respectively.

The feedback control for neighboring solutions are obtained as described below.

Let δt_1 be the shift in entry time as compared to t_1 of the nominal solution. This is known only during real flight and cannot be predicted earlier. The closed-loop guidance is initiated from initial time and the closed-loop load factor control, generated by the linear feedback law, is closely tracked. The closed-loop controls for some special regions are described below.

If the load factor n is unconstrained at t_1, then from time t_1 to $t_1 + \delta t_1$ the following control is used:

$$u_{\mathrm{CL}}(t) = u_N(t_1) + [t - t_1]\dot{u}_N(t_1)_- + G(t_1^-)\,\delta X(t) \tag{3}$$

Here u denotes the control vector $[n, \mu_b]^T$ and δX denotes the state perturbation from nominal. The suffix N denotes the reference values. The minus and plus subscripts in $(t)_-$ and $(t)_+$ denote values just to the left and right of (t).

If n encounters the constraint earlier than t_1, then $\delta t_1 < 0$ and the closed-loop control is given by

$$u_{\mathrm{CL}}(t) = u_N(t_1) + [t - t_1]\dot{u}_N(t_1)_+ + G(t_1^+)\,\delta X(t) \tag{4}$$

In the time zones $t_1^- < t < t_1^+, t_2^- < t < t_2^+$ and at times when the nominal and the closed-loop trajectories exhibit conflicting nature (one constrained and the other unconstrained), constant extrapolated gains are used. It can be easily shown that only second-order errors are introduced by this extrapolation.

The exit time from the constraint arc is decided by comparing the control n_{CL} below and the value of the nominal active control constraint:

$$n_{\mathrm{CL}}(t) = n_N(t_2) + [t - t_2]\dot{n}_N(t_2)_+ + G_n(t_2^+)\,\delta X(t) \tag{5}$$

When the former is less than the latter, the exit from the control constraint is made.

Small perturbations in each of the six states were used to obtain the gains. Test cases of closed-loop guidance were performed with different perturbations. The perturbed optimal control problem was re-solved (via BOUNDSCO) and results compared. The closed-loop solutions behave close to the exact solutions of the perturbed problem. A large perturbation at initial time (−20,000 ft in altitude) was made and the above closed-loop guidance strategy was carefully examined. The reference nominal trajectory used had an active structural limit of 13.4 g. Note that this value is much lower than expected for this type of missile and was used to illustrate the control limit phenomena. Figure 2 shows the nominal, perturbed optimal, and closed-loop load factor solutions. Figure 3 similarly exhibits the bank angle history. The control histories show the good performance of the closed-loop guidance in comparison to the re-solved TPBVP. Figure 4 shows a typical gain history ($\delta n/\delta E$) during the boost phase. The discontinuities in the gains at entry and exit points must be expected. Note the zero load factor gain along the control constraint. For larger initial perturbations it can be observed that the linearization is inaccurate, and this can be noticed with control jump at switching points, due to the increasing magnitude of second-order errors. A modified guidance scheme is required for cases where the nominal control becomes constrained "close" to a thrust switch from boost to sustain levels.[18]

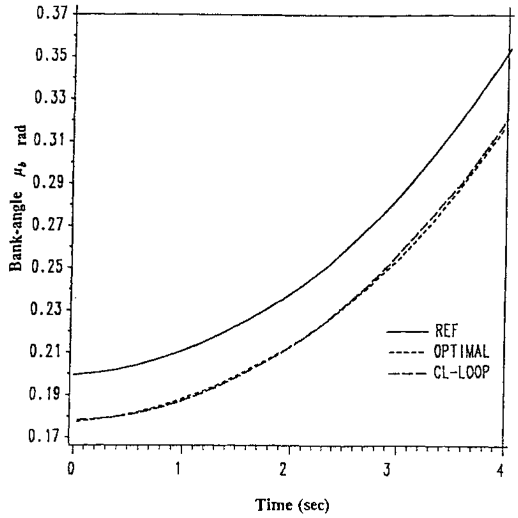

Fig. 3 Bank angle history: boost phase.

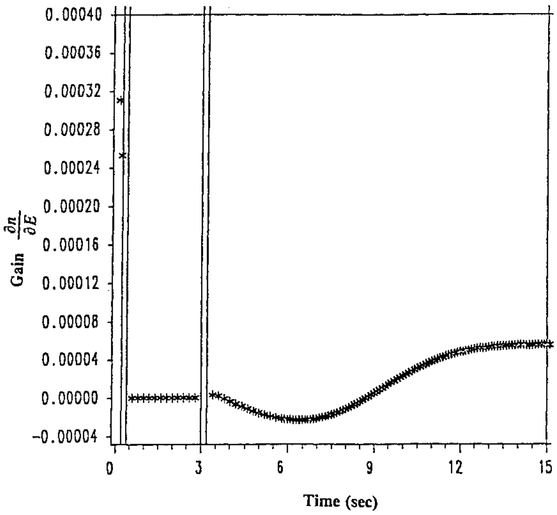

Fig. 4 Gain $\partial n / \partial E$ history: boost phase.

Midcourse Guidance

The midcourse guidance is initiated after the boost phase is completed, as discussed above. The reference solution is obtained as discussed earlier and the feedback gains are to be evaluated. The midcourse guidance phase has the longest duration. Transversal comparison[25–28] and other forms of performance augmentation have been employed during this phase.

Gain Evaluation

The gain evaluation for this phase of the guidance scheme is based on ideas of neighboring optimal control. The nominal minimum-time-to-intercept problem has a performance index $J = t_f$ and the following constraints:

$$\dot{X} = f(X, u, t), \qquad X(t_0) = X_0, \qquad \psi[X(t_f), t_f] = 0 \tag{6}$$

Here, X_0 is the vector of initial conditions and ψ is a vector function of final time and states at final time. Neighboring optimal control about the nominal trajectory is based on an "accessory minimum problem."[24] For brevity, the accessory minimum problem definition and the related matrices for the resulting Riccati method of gain evaluations[24] are not detailed in this paper. The neighboring optimal control law[24] is given by

$$\delta u(t) = G_1(t)\,\delta X(t) + G_2(t)\,d\psi(t) \tag{7}$$

where $\delta X(t)$ is a vector of state perturbation from nominal at any time and $d\psi(t)$ a vector representing a change in terminal conditions. The gains $G_1(t)$ and $G_2(t)$ are given by[24]

$$G_1 = -H_{uu}^{-1}\left[H_{ux} + f_u^T(\bar{S} - \bar{R}\bar{Q}^{-1}\bar{R}^T)\right] \tag{8}$$

$$G_2 = -H_{uu}^{-1} f_u^T \bar{R}\bar{Q}^{-1} \tag{9}$$

where H is the variational Hamiltonian and $\bar{S}$, $\bar{Q}$, and $\bar{R}$ are time-varying matrices.[24]

The nominal optimal control problem is a minimum-time problem and hence the final time also gets perturbed, and a good estimate[24] of the change in final time is given by

$$dt_f = K_1(t)\,\delta X(t) + K_2(t)\,d\psi(t) \tag{10}$$

where the gains $K_1(t)$ and $K_2(t)$ are given by

$$K_1(t) = -\left(\frac{m^T}{\alpha} - \frac{\hat{n}^T}{\alpha}\bar{Q}^{-1}\bar{R}^T\right) \tag{11}$$

$$K_2(t) = -\frac{\hat{n}^T}{\alpha}\bar{Q}^{-1} \tag{12}$$

Again, the vectors m, $\hat{n}$ (the "hat" is introduced to differentiate from load factor n) and the scalar α are time varying and are as defined in the Ref. 24 These gains are evaluated prior to flight and stored on-board. It is worthwhile to discuss briefly the choice for the terminal surface vector for the reference solution. One choice would be

$$\psi[X(t_f)] = \begin{bmatrix} x(t_f) - x_f \\ y(t_f) - y_f \\ h(t_f) - h_f \\ E(t_f) - E_f \end{bmatrix} \tag{13}$$

where the x_f, y_f, h_f, and E_f are fixed numbers decided by the optimal intercept point coordinate. Another choice would be

$$\psi[X(t_f)] = \begin{bmatrix} x(t_f) - [x_{T0} - V_T\cos(\lambda_T)t_f] \\ y(t_f) - [y_{T0} - V_T\sin(\lambda_T)t_f] \\ h(t_f) - h_f \\ E(t_f) - E_f \end{bmatrix} \tag{14}$$

where (x_{T0}, y_{T0}) is the target coordinate at the initial time. Here, V_T is the constant target velocity and λ_T the inertial bearing angle measured from the negative down-range axis of the missile in the clockwise direction.

The first choice is superior to the second in the sense that the feedback gains can be evaluated prior to flight. The second choice requires the target velocity and bearing angle as boundary conditions for the matrix differential equations, which are then integrated backward in time to obtain the feedback gains. However, the advantage of the second choice is that for a target moving at constant velocity along a straight line, $d\psi(t) = 0$, even when state perturbations are made during the flight. Hence, the second method is advantageous for nonmaneuvering targets.

If the gains are scheduled with a clock time comparison, i.e., comparison between the states at the same time, then for the case when $dt_f > 0$ one can run out of gains. The perturbation dt_f can be calculated from Eq. (10) by substituting $d\psi$ as follows:

$$d\psi = \left(\frac{\partial\psi}{\partial X}\delta X + \frac{\partial\psi}{\partial t}dt_f\right)_{t_f} \tag{15}$$

Once dt_f and $d\psi$ are obtained, the closed-loop control vector can be evaluated by using Eq. (7).

The feedback gains can also be evaluated by using the method of perturbation as in Eq. (2) during the boost phase guidance. The gains via the perturbation method and gains obtained from Riccati solutions show little difference. Figure 5 gives a good comparison for a sample gain done by the two methods. The perturbation method becomes attractive for the case when H_{uu} becomes singular. This occurs when the missile flight-path angle and heading angle at final

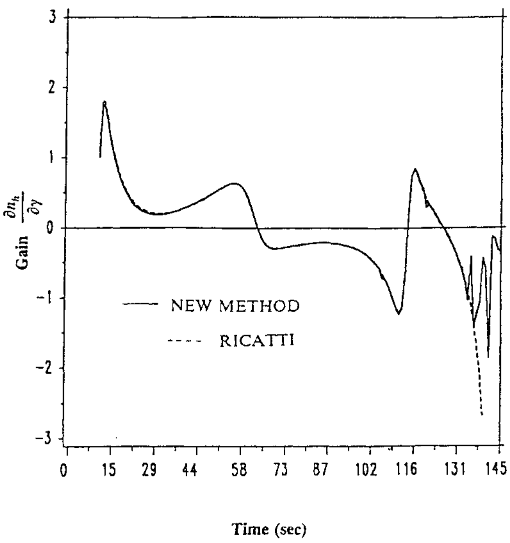

Fig. 5 Gain $\partial n_h/\partial\gamma$ history: midcourse phase.

time are unspecified; the corresponding adjoint variables are zero at final time.[17] When this happens, the accessory minimum problem is singular at t_f and the associated matrix Riccati differential equation is invalid. It has been found that the problem can be "regularized" with penalty terms.[17]

Transversal Comparison

Since the problem has no explicit dependence on time and to prevent the "running out" of gains, the time to go $(t_f - t)$ is really the important time and the gains are "keyed" to it.[24,28] The difficulty of unbounded gains is avoided if the gain tables are entered at an index time determined so that the time to go on the closed-loop neighboring path is the same as the time to go for the nominal path. This is the same idea as performance index to go as suggested by Kelley.[25]

Since the time to go t_f' is to be the same on the neighboring and nominal paths, one has

$$t_f' = t_f - t = t_f^N - t_I \tag{16}$$

where t_f is the estimated final time of the neighbor, t is the clock time, t_f^N is the terminal time of the nominal solution, and t_I is the desired index time. The perturbation in final time $(t_f - t_f^N)$ is estimated from Eq. (10) so that the implicit equation for the index time[28] is

$$t - t_I = \frac{K_1(t_I)[X(t) - X_N(t_I)] + K_2(t_I)[d\psi]}{1 + K_1(t_I)\dot{X}_N(t_I)} \tag{17}$$

One starts the iterative techniques of solution of Eq. (17) with the estimate of $t_I = t$. The gains K_1, K_2, and $\dot{X}_N$ are evaluated at this index time. Here, $d\psi$ may or may not be a function of time to go. A new t_I is evaluated using Eq. (17). This procedure is repeated until t_I converges.[28] In other words, Eq. (17) may be rewritten as $t_I = g(t_I)$ [by adding $-t$ to both sides of Eq. (17) and multiplying by -1] and a fixed-point iteration performed to obtain a root of this equation. This method always converged in two to three iterations to the required tolerance.

An explicit formula for the closed-loop control assuming smooth controls and state is given by

$$u(t) = u_N(t_I) + G_1(t_I)[X(t) - X_N(t_I)] + [G_1(t_I)\dot{X}_N(t_I) + \dot{u}_N(t_I)][t - t_I] + G_2(t_I)\,d\psi \tag{18}$$

The closed-loop control is used to simulate a real-time flight situation of the missile with target maneuvers and state perturbations. Indexing with time to go for this problem shows better simulation results, particularly during the later period of flight. Simulation was performed for an aggressive target, and a run-away target, and the guidance scheme performs better for both cases when index time is used.

Performance Augmentation

It has been observed that for the aforementioned reference solution, the closed-loop guidance works well only for small $d\psi$ [using ψ from Eq. (13)]. If the target tries to turn away and the missile ends up in a tail chase, then $d\psi$ increases with time and the neighboring optimal scheme fails. The same is true if ψ defined in Eq. (14) is used. However, if the target continues to be aggressive with only small perturbations from the nominal (anticipated) target trajectory, the closed-loop guidance works effectively.

In order to reduce the magnitude of $d\psi$ for all possible target maneuvers, the concept of aiming at the center of attainability of the target is visualized. The center of attainability of the target is defined as the average value of the maximum range in the forward direction and the maximum range in the backward direction on the symmetric axis of the attainability set of the target moving in the horizontal plane.

The target is assumed to maneuver only in the horizontal plane with a constant velocity enabling simplified target dynamics:

$$\dot{x}_T = -V_T\cos(\lambda_T + \chi_T) \tag{19}$$

$$\dot{y}_T = -V_T\sin(\lambda_T + \chi_T) \tag{20}$$

$$\dot{\chi}_T = \frac{n_T g}{V_T} \tag{21}$$

where (x_T, y_T) are the target coordinates and V_T, χ_T, and n_T are the target velocity, heading angle (measured with respect to the nominal target direction), and the turning load factor, respectively. The term λ_T is as defined earlier and g is the acceleration due to gravity. The load factor in Eq. (21) is assumed to be the control variable for the target with an upper bound in magnitude.

It can be easily derived that the distance of the center of attainability of the target from its current position along the line of the instantaneous velocity direction is given by $p(t_f')$:

$$p(t_f') = \begin{cases} at_f' + b\,\sin(ct_f') & \text{if} \quad t_f' < \dfrac{\pi V_T}{g n_{T,\max}} \quad (22) \\ \dfrac{\pi V_T^2}{2 g n_{T\max}} & \text{if} \quad t_f' \ge \dfrac{\pi V_T}{g n_{T,\max}} \quad (23) \end{cases}$$

where

$$a = \tfrac{1}{2}V_T, \qquad b = V_T^2/2gn_{T,\max}, \qquad c = gn_{T,\max}V_T$$

Here, $n_{T,\max}$ is the maximum turn-rate load factor, and V_T is the constant velocity of the target to be estimated by the missile, and t_f' is the time to go for the missile–target intercept.

This definition of center of attainability is simply an ad hoc scheme to construct a new nominal terminal surface for the missile such that the perturbation $d\psi$ is kept within bounds for all possible target maneuvers in the horizontal plane. Alternate statistical models for the pseudotarget behavior may be used.[29] The new nominal minimum-time optimal intercept problem has the following $d\psi$:

$$\psi[X(t_f)] = \begin{bmatrix} x(t_f) - [x_{T0} - p(t_f')\cos(\lambda_T)] \\ y(t_f) - [y_{T0} - p(t_f')\sin(\lambda_T)] \\ h(t_f) - h_f \\ E(t_f) - E_f \end{bmatrix} \tag{24}$$

Time to go t_f' is as defined earlier. Since t_f^N for the reference solution is known, $d\psi = F(t_I)$. This functional relationship can be seen from Eqs. (22) and (24). The index time t_I is solved by iterating Eq. (17) until convergence to the desired tolerance. Along with the converged value of t_I, we obtain the corresponding values of t_f', $p(t_f')$, and $d\psi$, which are then used for evaluating the closed-loop control.

The closed-loop guidance was simulated using the gains derived for the above nominal minimum-time intercept problem. Simulation of target behavior was first performed with a target moving with a maximum velocity of 700 ft/s (approximately half the velocity of the missile velocity at launch), as shown in Fig. 6. The time to go, required to obtain the center of attainability of the target and

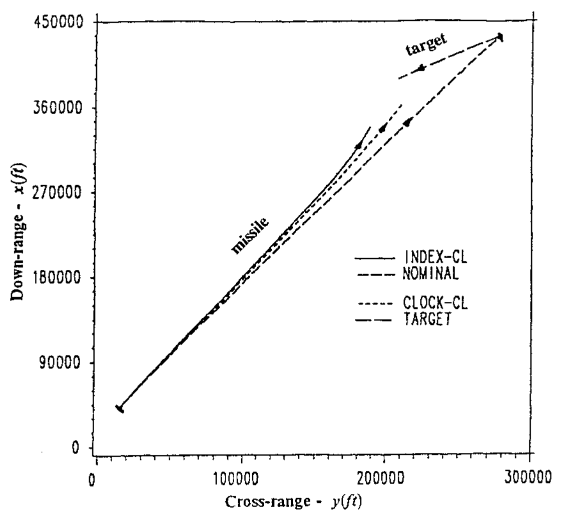

Fig. 6 An x–y projection of midcourse trajectory: aggressive target.

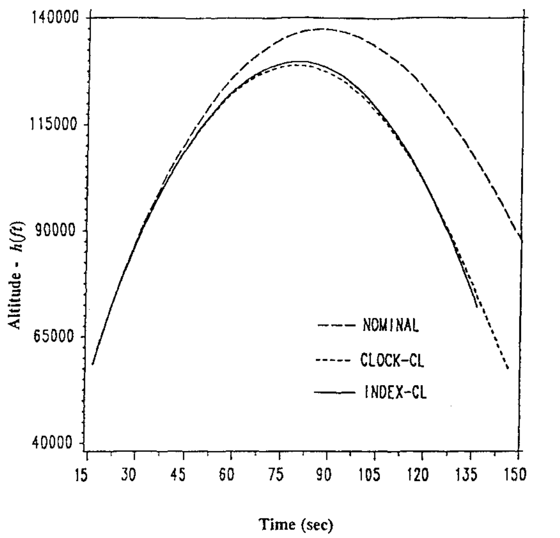

Fig. 7 Altitude time history of midcourse trajectory: aggressive target.

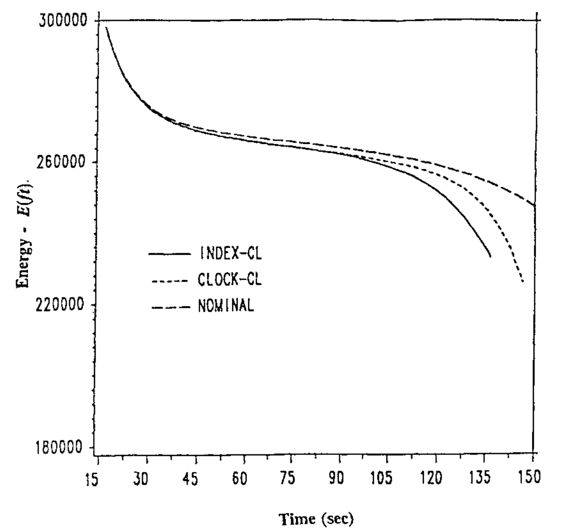

Fig. 8 Energy time history of midcourse trajectory: aggressive target.

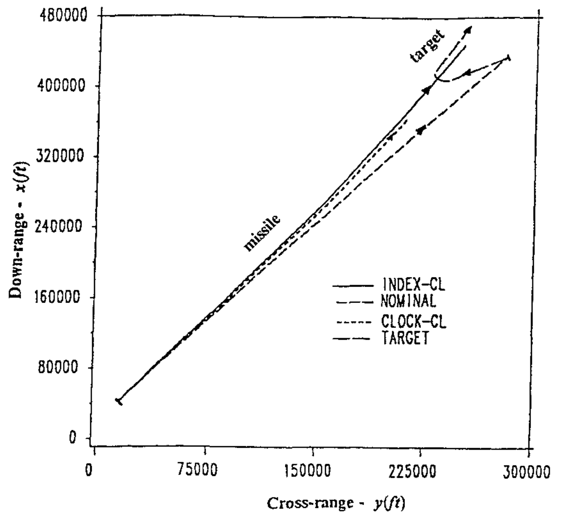

Fig. 9 An x–y projection of midcourse trajectory: run-away target.

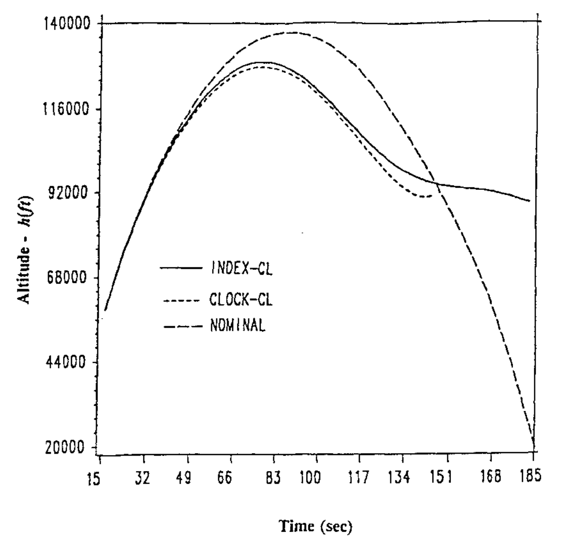

Fig. 10 Altitude time history of midcourse trajectory: run-away target.

for gain indexing, is obtained in a few iterations by the fixed-point iteration. The closed-loop guidance worked very well, with the index time gain scheduling showing better performance than clock time comparison, especially during the latter phase of flight. Figures 6–8 show the nominal and closed-loop trajectories (index and clock time) for position states and energy.

Figure 6 indicates a better heading direction for the AAM using performance-index-to-go transversal comparison. This is due to the fact that the index timing uses the gains at the same time to go of the nominal, and these gains are of higher magnitude than the gains at clock time. Comparison with the re-solved TPBVP of the perturbed problem is unfair, since it does not convey the instantaneous guidance behavior of AAM due to target maneuver. It would only show the optimal trajectory for intercepting the final prescribed target coordinate. The guidance scheme breaks down, due to increasing gains and increasing magnitudes of $d\psi$ and δX, at about 131.47 s with an estimated time to go of 18 s indicating an approximate overall time of 154.47 s. This final time is only 4.47 s more than the optimal trajectory, which intercepts the target at 150 s. "Appropriate" hand-over to a terminal guidance scheme like PN is required. An approximate bound on slopes and magnitudes of control efforts and time to go may be considered as the criteria for the hand-over to terminal guidance. This paper does not deal with issues of hand-over from midcourse to terminal guidance.

A second target maneuver used for simulation was as follows: The target travels along a straight line in the horizontal plane with a constant maximum velocity of 700 ft/s for the first 80 s, then turns around with maximum lateral acceleration in the horizontal plane in a direction such that the LOS angle from the target is increased until it is 180 deg. The target then flees in a straight line, as shown in Fig. 9. The guidance scheme breaks down at about 182.47 s with time to go of about 21.2 s. The altitude management of the missile to conserve potential energy for the end game is seen in Fig. 10. The energy conservation for the long flight can be clearly seen in Fig. 11.

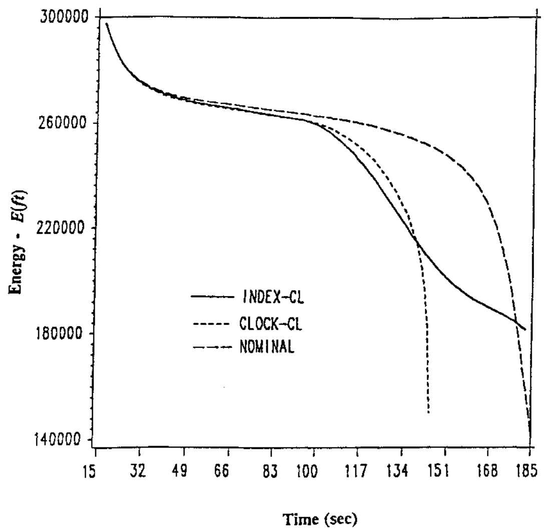

Fig. 11 Energy time history of midcourse trajectory: run-away target.

Simulation results against more sophisticated target models utilizing differential game theory and singular perturbation techniques have been performed, and the near-optimal scheme was found to perform satisfactorily.[18] A new enhanced composite midcourse guidance scheme utilizing PN in the horizontal plane and neighboring optimal control in the vertical plane, christened as half-PN, resulted from this study.[18] The major attraction of half-PN, is the reduction in storage space in the missile computer by nearly half in comparison to a purely neighboring optimal guidance. This is because the gains for the horizontal load factor control are not required. A purely PN approach for the entire flight is not feasible since the missile "runs out of steam" in a short time[18] (25 s) due to high drag at low altitudes.

Terminal Guidance

Pure proportional navigation (PPN) in three dimensions[30] is used for the AAM for terminal guidance. This is defined as follows: Construct the lead-angle plane containing the velocity vector of the missile at a given time and the LOS to the target. The dynamics of the missile and target may induce a nonzero rate of change of the LOS. The rate of change is re-solved into two orthogonal components: one in the lead-angle plane, normal to the LOS of magnitude $\dot{\sigma}_u$, and the other normal to the lead-angle plane of magnitude $\dot{\sigma}_v$. The inertial acceleration vector of the missile a_m can be composed as

$$a_m = V\dot{\hat{e}}_\gamma + g(-D/W - \sin\gamma)\hat{e}_\gamma \tag{25}$$

where $\hat{e}_\gamma$ is the unit vector along the velocity and $\dot{\hat{e}}_\gamma$ is given by

$$\dot{\hat{e}}_\gamma = \dot{\gamma}_u\hat{e}_{\gamma u} + \dot{\gamma}_v\hat{e}_{\gamma v} \tag{26}$$

where $\hat{e}_{\gamma u}$ and $\hat{e}_{\gamma v}$ are unit vectors normal to velocity in and out of the lead-angle plane, respectively.

The quantities $\dot{\sigma}_u$ and $\dot{\sigma}_v$ can be obtained with information on the target position, the rate of change of position and the missile's own position and velocities. The PPN law states that $\dot{\gamma}_u = P_1\dot{\sigma}_u$ and $\dot{\gamma}_v = P_2\dot{\sigma}_v$. The above law is then related to the specific model and the control variables n_v and n_h are evaluated after suitable transformations. The parameters P_1 and P_2 are proportionality constants to be designed, whose range varies from 2 to 5 in general. The usual practice is to consider $P_1 = P_2$ and to assign a constant value depending on the model and objective. It has been shown[24] that a value of 3 for the constants is in fact optimal in the sense of minimizing a weighted sum of miss distance and control (acceleration) cost for a simple intercept problem with a nonmaneuvering target. Classical true-proportional navigation (TPN) can also be used instead of PPN.

It is to be noted that the acceleration in the velocity direction $\hat{e}_\gamma$ is fixed for a given n_v and n_h and hence not directly controlled by any criteria. The acceleration command along the direction of the missile velocity is obtained from the locus of all possible axial accelerations allowed for the missile model for prescribed lateral acceleration. This is due to the fact that there exists only two controls to match three acceleration components in the missile guidance. The lack of an axial control-like thrust during the end game shows the reduction in controllability of the missile to choose any acceleration vector. The terminal guidance is basically used in this study to check whether the midcourse guidance behaved satisfactorily. It is observed that the PN scheme employing PPN or TPN is efficient in terms of time and energy at the intercept for the aggressive target. The end of midcourse guidance terminates with the missile and target closely aligned in a collision course with a small time to go. Simulation of terminal guidance using PPN or TPN for a run-away target shows low energy at intercept time compared to the reference trajectory. The extended time of flight for the missile chasing the run-away target, supplemented by the fact that the thrust is zero, makes the lower energy at interception only obvious.

Any attempt to improve performance by linearizing the missile model[31] using a Brunovsky canonical form[32] and to solve a linear quadratic control problem to optimize final energy and miss distance is rendered useless due to the absence of the third axial control. This simply implies the fact that the acceleration command obtained from the linear control problem may not be feasible.

Since the midcourse guidance of the missile chased the center of attainability of the target, the terminal guidance can be smoothly handed over using PN chasing the pseudotarget. The time to go for intercept during the midcourse guidance was evaluated by suitable linearization and prediction of the final time of the neighboring trajectory. For the terminal guidance, one predicts the time to go to hit the target by a simple three-dimensional collision course. The results show marginal improvement over the classical PN schemes chasing the target. Figures 12 and 13 show the horizontal projection

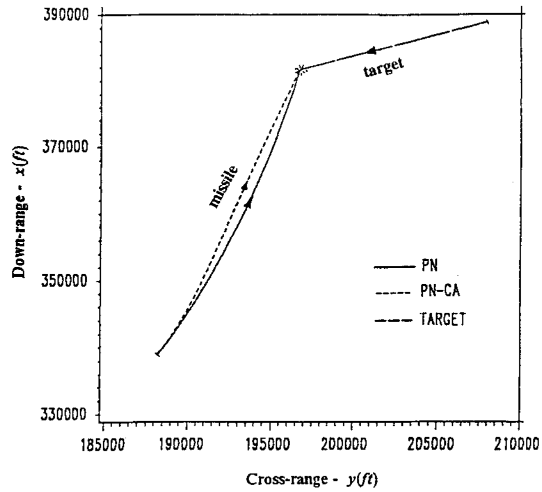

Fig. 12 An x–y projection, terminal phase: aggressive target.

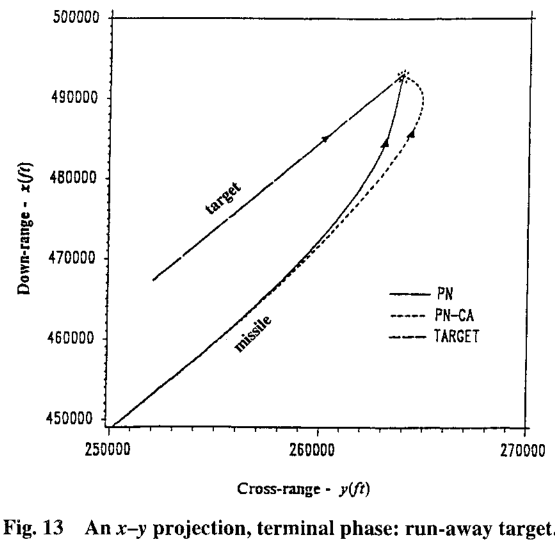

Fig. 13 An x–y projection, terminal phase: run-away target.

of the missile by using the above method for aggressive and run-away targets, respectively. The pseudotarget chasing for both the simulations has a distinct advantage of the missile chasing a target with slower dynamics. The final energy is close to optimal for the aggressive target homing, but for the run-away target chasing, the final energy is much below nominal.

Attempts to improve this final energy at intercept were made using schemes like drag resolution and altitude shaping.[18] Only little improvement over the classical methods has been obtained by any of the suggested methods. Thus one concludes that the long flight times associated with the target model and the limited energy of the missile are the fundamental reasons for the same. Hence, the time of launch of the missile, optimal thrust-to-weight ratio of the missile, the targets of interest, no-escape envelopes, etc., should be studied in detail in order to use the three-phase guidance scheme successfully in real combat scenarios.

Conclusions

The boost phase of the guidance shows switching structures of the closed-loop trajectory behaving near optimally when compared to the optimal solutions of the perturbed problem. The closed-loop entry and exit switching times and the state and control histories show near-optimal nature even for large state perturbations. The method suggested in this paper to evaluate gains with TPBVPs resolved only at initial time has not been previously reported in the literature.

The midcourse guidance employs index time transversal comparison. Performance was improved by using a pseudotarget based on a center of attainability concept and by restructuring the nominal intercept reference solutions to take into account large changes in end conditions. The performance of the midcourse guidance with these modifications is extremely good. Simulation results against more sophisticated target models have been performed and the near-optimal scheme was found to perform satisfactorily. A new enhanced composite guidance scheme utilizing PN in the horizontal plane and neighboring optimal control in the vertical plane resulted from this study.

Classical PN in three dimensions has been used for terminal guidance. Only marginal improvement in final energy at intercept has been achieved by using ideas of pseudotarget chasing, altitude shaping, and drag resolution.

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References

[1]Locke, A. S., *Guidance, a Volume in the Series: Principles of Guided Missile Design*, edited by G. Merrill, Van Nostrand, Princeton, NJ, 1955.

[2]Yuan, C. L., "Homing and Navigational Courses of Automatic Target-Seeking Devices," *Journal of Applied Physics*, Dec. 1948, pp. 1122–1128.

[3]Newell, H. E., "Guided Missile Kinematics," Naval Research Laboratory, Rept. R-2538, Washington, DC, May 1945.

[4]Spitz, H., "Partial Navigation Courses for a Guided Missile Attacking a Constant Velocity Target," Naval Research Laboratory, Rept. R-2790, Washington, DC, March 1945.

[5]Murtaugh, S. A., and Criel, H. E., "Fundamentals of Proportional Navigation," *IEEE Spectrum*, Vol. 3, Dec. 1966, pp. 75–78.

[6]Kishi, F. H., "Optimal and Sub-Optimal Designs of Proportional Navigation Systems," *Recent Advances in Optimization Techniques (Proceedings of the April 1965 Symposium)*, edited by A. Lavi and T. P. Vogl, Wiley, New York, 1965.

[7]Bryson, A. E., "Linear Feedback Solutions for Minimum Effort Interception, Rendezvous, and Soft Landing," *AIAA Journal*, Vol. 3, No. 8, pp. 1542–1544.

[8]Bryson, A. E., "Application of Optimal Control Theory in Aerospace Engineering," *Journal of Spacecraft and Rockets*, Vol. 4, No. 5, 1967, pp. 545–553.

[9]Kreindler, E., "Optimality of Proportional Navigation," *AIAA Journal*, Vol. 11, No. 6, 1973, pp. 878–880.

[10]Cottrell, R. G., "Optimal Intercept Guidance for Short-Range Tactical Missiles," *AIAA Journal*, Vol. 9, 1971, pp. 1414, 1415.

[11]Cheng, V. H. L., and Gupta, N. K., "Advanced Midcourse Guidance for Air-to-Air Missiles," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 2, 1986, pp. 135–142.

[12]Menon, P. K. A., and Briggs, M. M., "A Midcourse Guidance Law for Air-to-Air Missiles," *Proceedings of Guidance, Navigation, and Control Conference*, AIAA Washington, DC, 1987 (AIAA-Paper 87-2509).

[13]Kelley, H. J., "Aircraft Maneuver Optimization by Reduced-Order Approximation,"*Control and Dynamic Systems*, edited by C. T. Leodes, Vol. 10, Academic, New York, 1973, pp. 131–178.

[14]Ardema, M. D., "Singular Perturbations in Flight Mechanics," NASA TMX-62, 1974 (revised July 1977).

[15]Calise, A. J., "On the Use of Singular Perturbation Methods in Variational Problems," *Proceedings of Joint Automatic Control Conference*, Ohio State Univ., June 1973, Paper 7–4, pp. 184–192.

[16]Katzir, S., "Optimal and On-Board Near-Optimal Midcourse Guidance," Ph.D. Dissertation, Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, Nov. 1988.

[17]Kumar, R. R., Seywald, H., Cliff, E. M., and Kelley, H. J., "3-D Air-to-Air Missile Trajectory Shaping Study," *Proceedings of AIAA Guidance, Navigation, and Control Conference*, AIAA, Washington, DC, 1989 (Paper No. 89-8337).

[18]Kumar, R. R., "Optimal and Near-Optimal Medium Range Air-to-Air Missile Guidance Against Maneuvering Targets," Ph.D. Dissertation, Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, June 1989.

[19]Kelley, H. J., "Guidance Theory and Extremal Fields," *IRE Transactions on Automatic Control*, Vol. AC-7, 1962, pp. 75–82.

[20]Hymas, C. E., Cavin, R. K., and Colunga, D., "Neighboring Extremals for Optimal Control Problems," *AIAA Journal*, Vol. 11, No. 8, 1973, pp. 1101–1109.

[21]Pesch, H. J., "Real-Time Computation of Feedback Controls for Constrained Optimal Control Problems," Report No. 12, Technical University of Munich, Germany, 1987.

[22]Bulirsch, R., "Die Mehrzielmethode zur numerischen Losung von nichtlinearen Randwertproblemen und Aufgaben der optimalen Steuerung," *Lehrgang Flugbahnoptimierung Carl-Cranz-Cessellschaft e.v.*, Oct. 1971.

[23]Visser, H. G., "Energy Management of Three-Dimensional Minimum-Time Intercept," Ph.D. Dissertation, Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, Aug. 1985.

[24]Bryson, A. E., and Ho, Y. C., *Applied Optimal Control*, Hemisphere, New York, 1975.

[25]Kelley, H. J., "An Optimal Guidance Approximation Theory," *Transactions of IEEE*, Vol. AC-9, 1964, pp. 375–380.

[26]Powers, W. F., "A Method for Comparing Trajectories in Optimum Linear Perturbation Guidance Schemes," *AIAA Journal*, Vol. 6, No. 12, 1968, pp. 2451, 2452.

[27]Powers, W. F., "Techniques for Improved Convergence in Neighboring Optimal Guidance," *AIAA Journal*, Vol. 8, No. 12, 1970, pp. 2235–2241.

[28]Speyer, J. L., and Bryson, A. E., "A Neighboring Optimum Feedback Control Scheme Based on Estimated Time-to-Go with Applications to Re-Entry Flight Paths," *AIAA Journal*, Vol. 6, No. 5, 1968, pp. 769–776.

[29]Cloutier, J. R., Evers, J. H., and Feeley, J. J., "An Assessment of Air-to-Air Missile Guidance and Control Technology," *Proceedings of the ACC*, Inst. of Electrical and Electronics Engineers, New York, 1988, pp. 133–142.

[30]Adler, F. P., "Fundamentals of Proportional Navigation," *Journal of Applied Physics*, Vol. 27, No. 5, 1956, pp. 500–507.

[31]Menon, P. K. A., "Short-Range Nonlinear Feedback Strategies for Aircraft Pursuit-Evasion," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 1, 1989, pp. 27–32.

[32]Meyer, G., Su, R., and Hunt, L. R., "Applications of Nonlinear Transformations to Automatic Flight Control," *Automatica*, Vol. 20, 1984, pp. 103–107.